

A Formal Approach towards Safe and Stable Schedule Synthesis in Weakly Hard Control Systems

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Introduction: Plant-Control Closed-Loop Systems



An autonomous car

A dynamical system like a physical process
(Plant)

Introduction: Plant-Control Closed-Loop Systems

Speed sensor measures
the car's actual speed



An autonomous car

Introduction: Plant-Control Closed-Loop Systems



Cruise controller of the car



Speed sensor measures
the car's actual speed



An autonomous car

Stabilizing Feedback Controller

Introduction: Plant-Control Closed-Loop Systems



Cruise controller of the car

*Compares actual speed
with the reference speed*



Speed sensor measures
the car's actual speed



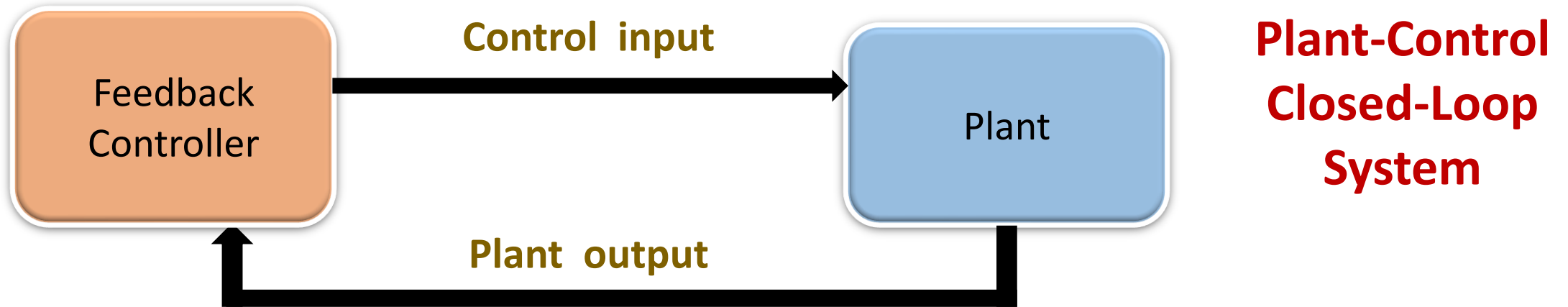
An autonomous car



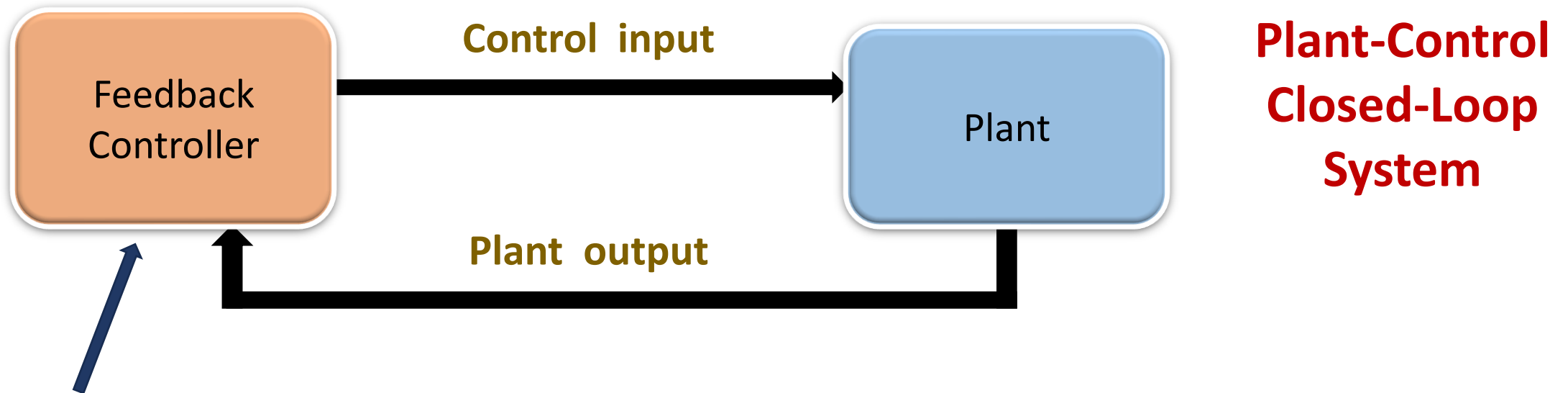
Control Input:

Adjusts the throttle force

Introduction: Plant-Control Closed-Loop Systems

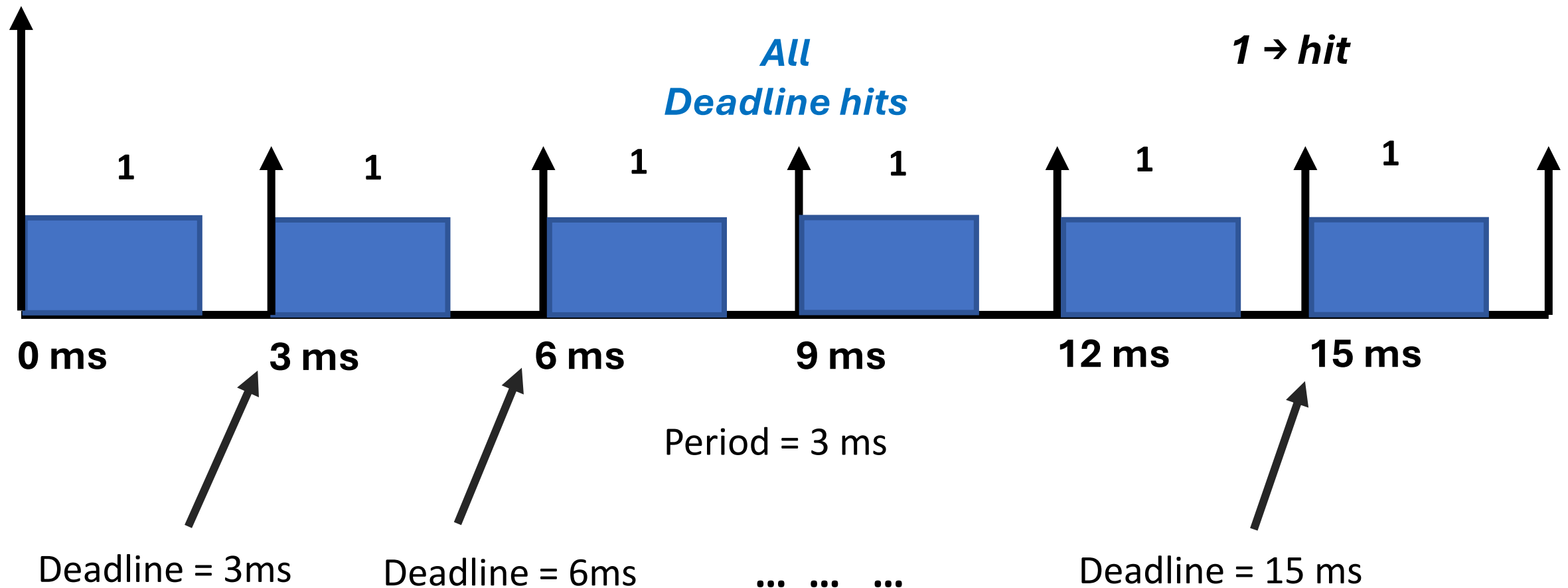


Introduction: Plant-Control Closed-Loop Systems

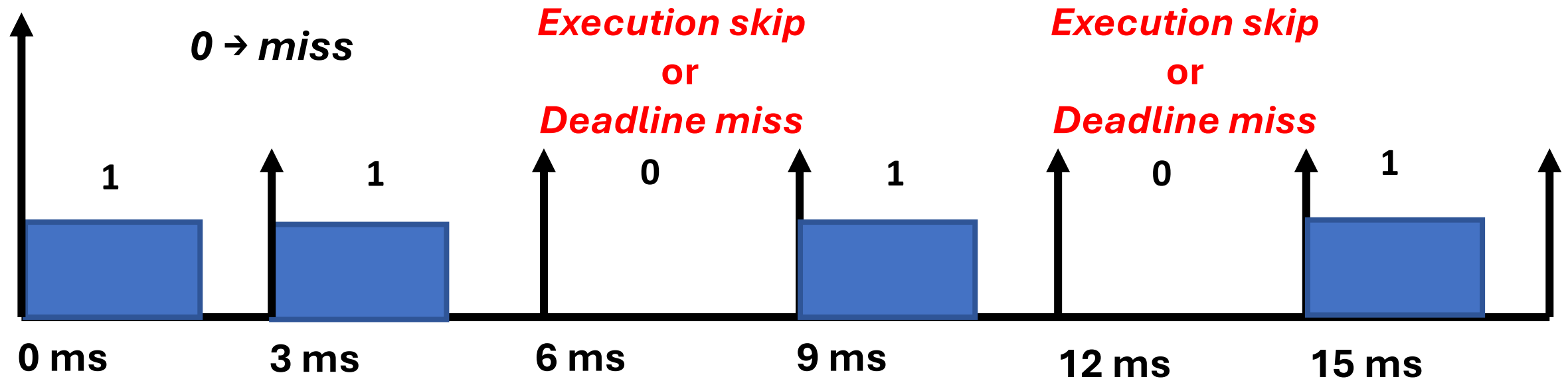


- Discrete-time controller executed as a **software control task** in the embedded processor.
- Control Task: periodic real-time task, should complete execution before its **deadline**.
- This is the task's **hard real-time requirement** to satisfy.

Introduction: Scheduling Control Task in Hard Real-Time Setting



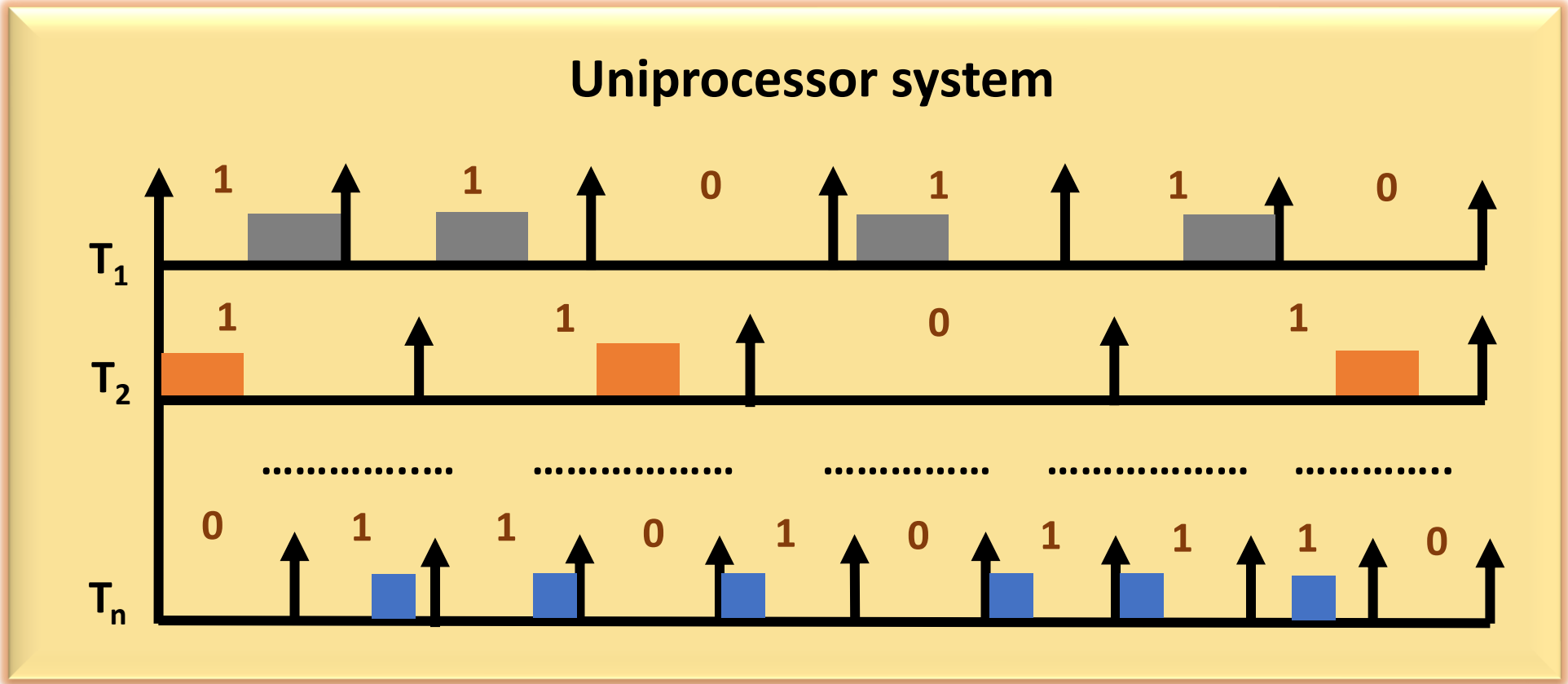
Introduction: Scheduling Control Task in Weakly Hard Setting



(4, 6)-firm weakly hard constraint

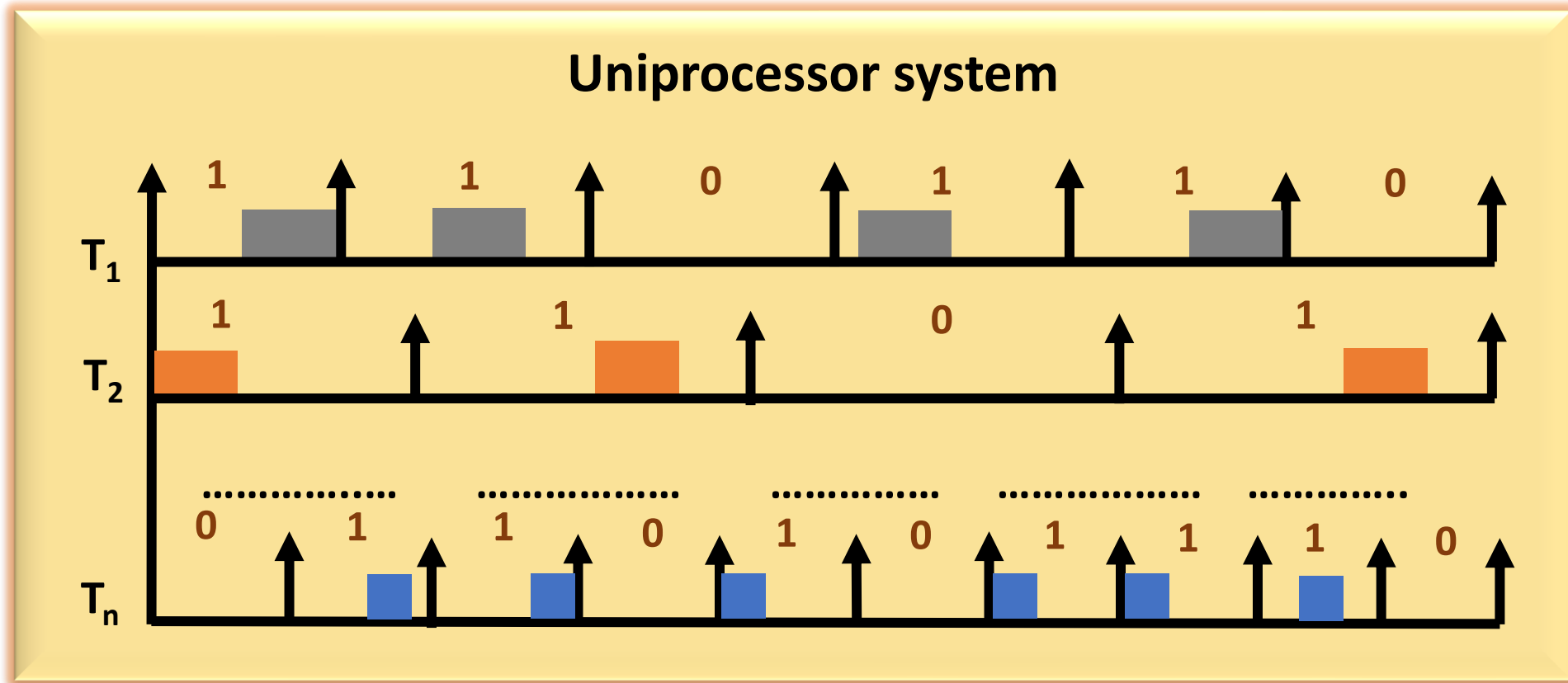
Deadline hit-miss pattern: **Control Execution Sequence (CES)** e.g., **110101**

Scheduling Multiple Weakly Hard Control Tasks in a Processor



n weakly hard control tasks in a processor corresponding to the n controllers

Motivation of the Work



Does such a schedule ensure the underlying **stability** of the control systems ?

Does such a schedule ensure the underlying **safety** of the control systems ?

Limitations of Existing Methods and Contributions of this Work

Limitations:

- ❑ In the context of *schedulability*: Existing methods focus **either** on *stability*^{1,2} **or** *safety*^{3,4}.
- ❑ Blend: (stability, safety, schedulability) is **missing**.
- ❑ Schedule ensures **safety only over a bounded time horizon**^{3,4}.

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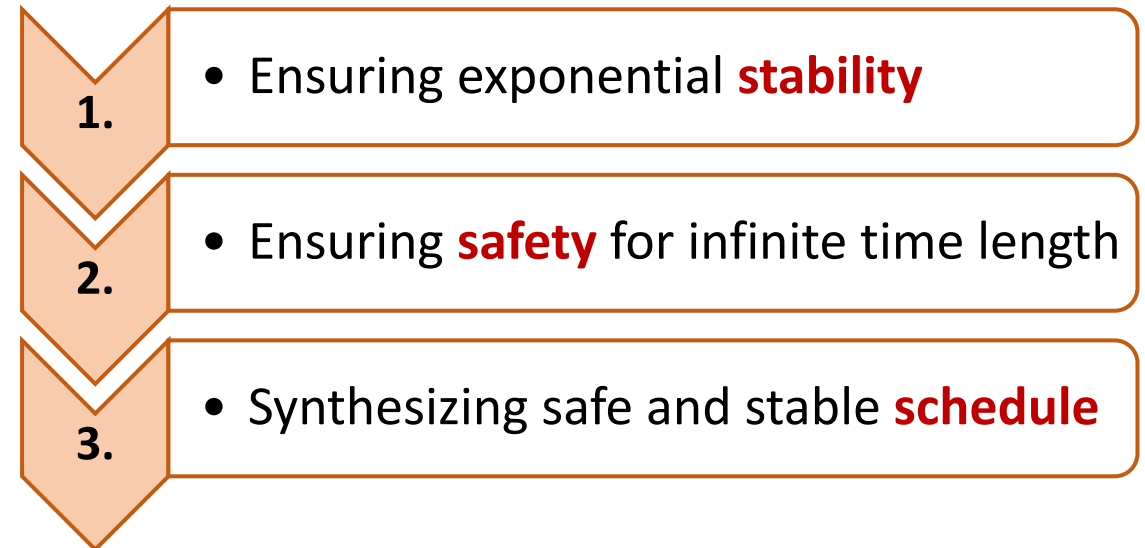
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Contributions:

Addressing (*stability, safety, schedulability*) for the **first time** → **schedule** ensures **stability and safety** over an **unbounded time horizon**.

Stepwise exploring the 3 aspects:



Step 1: Ensuring Stability

Ensuring Stability

(l, ϵ) -exponential stability criterion

Every l -length ratio of norm $\|x\|$ decreases by damping ratio of ϵ .

$$\text{i.e., } \frac{\|x[k+l]\|}{\|x[k]\|} < \epsilon$$

Ensuring Stability

Inputs

Settling time (S)
+
Reference value (ξ)
+
Maximum disturbance
allowed at input (δ)

(l, ϵ) -exponential stability criterion

Every l -length ratio of norm $\|x\|$
decreases by damping ratio of ϵ .

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Ensuring Stability

Inputs

Settling time (S)
+
Reference value (ξ)
+
Maximum disturbance
allowed at input (δ)



Compute (l, ϵ) -exponential stability criterion

$$N = \left\lceil \frac{S}{h} \right\rceil,$$

$$l = \left\lceil \frac{N}{f} \right\rceil,$$

$$\epsilon = \left(\frac{\xi}{\xi + \delta} \right)^{\frac{1}{f}}$$

h : sampling period
 f : tuning parameter

(l, ϵ) -exponential stability criterion

Every l -length ratio of norm $\|x\|$
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Ensuring Stability

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Compute (l, ϵ) -exponential stability criterion



Compute minimum control execution rate r

(l, ϵ) -exponential stability criterion

Every l -length ratio of norm $\|x\|$
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$$\text{i.e., } \frac{\|x[k+l]\|}{\|x[k]\|} < \epsilon$$

$$\beta = \frac{\ln\left(\frac{1}{\epsilon}\right)}{l \times h},$$

$$r = \frac{2 \ln(\beta) + \ln(\chi_0)}{\ln(\chi_0) - \ln(\chi_1)}$$

χ_0, χ_1 : *spectral radii of
the open loop and closed
loop dynamics matrices*

Ensuring Stability

Inputs

Settling time (S)
+
Reference value (ξ)
+
Maximum disturbance
allowed at input (δ)

Compute (l, ϵ) -exponential stability criterion

Compute minimum control execution rate r

Compute *stable* (M, K) -firm constraint

$$M = \lceil r \times l \rceil , \\ K = l$$

(l, ϵ) -exponential stability criterion

Every l -length ratio of norm $\|x\|$
decreases by damping ratio of ϵ .

$$\text{i.e., } \frac{\|x[k+l]\|}{\|x[k]\|} < \epsilon$$

Ensuring Stability

Inputs

Settling time (S)
+
Reference value (ξ)
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Maximum disturbance
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Compute (l, ϵ) -exponential stability criterion



Compute minimum control execution rate r



Compute *stable* (M, K) -firm constraint



Ensuring system's *exponential stability*

(l, ϵ) -exponential stability criterion

Every l -length ratio of norm $\|x\|$
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Ensuring Stability

Inputs

Settling time (S)
+
Reference value (ξ)
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Maximum disturbance
allowed at input (δ)



Compute (l, ϵ) -exponential stability criterion



Compute minimum control execution rate r



Compute *stable* (M, K) -firm constraint



Ensuring system's *exponential stability*



System's state converges to equilibrium
point with *exponential decay rate*

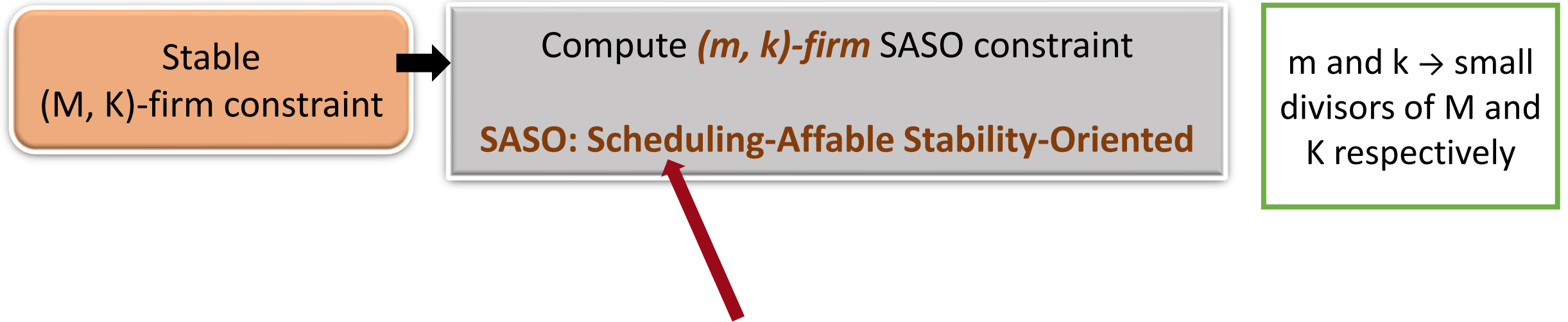
(l, ϵ) -exponential stability criterion

Every l -length ratio of norm $\|x\|$
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$$\text{i.e., } \frac{\|x[k+l]\|}{\|x[k]\|} < \epsilon$$

$$\|x[k]\| \rightarrow 0$$

SASO Constraint



- *Scheduling affable*: SASO constraints speed up the scheduling process.
- SASO constraints enhance the control performance:
CESs following SASO constraints avoid scenarios of missing deadlines consecutively often.

Step 2: Ensuring Safety

Criterion for Safety

Bounded deviation from ideal behavior or nominal trajectory

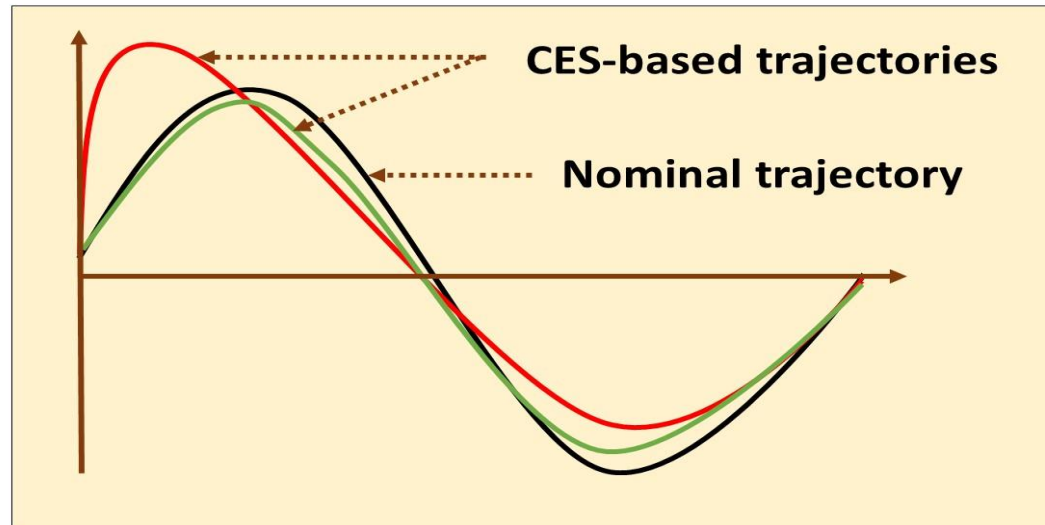
Criterion for Safety

Bounded deviation from ideal behavior or nominal trajectory :

- **Nominal trajectory (N):** State evolution trajectory for all deadlines met, i.e., pattern '111...' .
- **CES-based trajectory (C_p):** State evolution trajectory for deadline hit-miss pattern (CES), e.g., 11010.

← ideal behavior of the system

← system's behavior with deadline misses

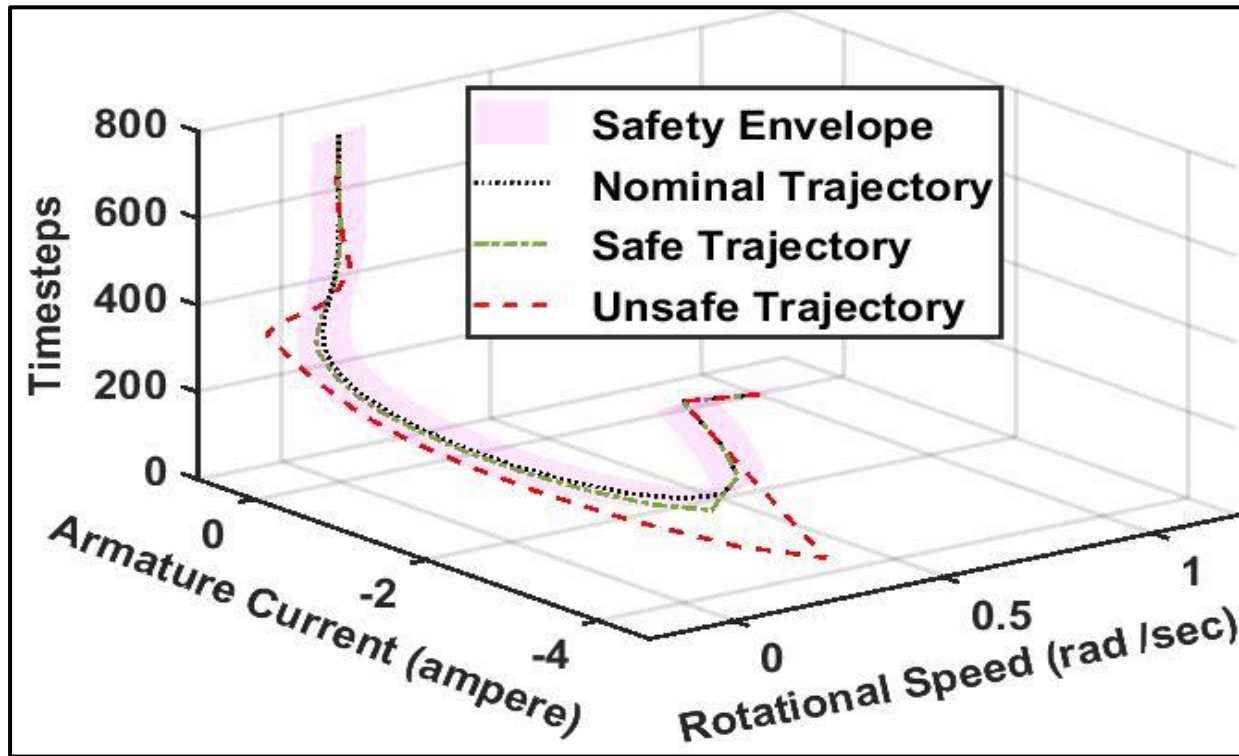


Criterion for Safety

Bounded Deviation: The deviation must be bounded, i.e., $dis(N, C_p) \leq d^{safe}$, where d^{safe} is the safety bound.

Deviation of C_p from N :
 $dis(N, C_p)$

(measured in terms of
Euclidean distance)



DC motor speed control system

Ensuring Safety

Input

(m, k)-firm SASO
constraint

Ensuring Safety

Input

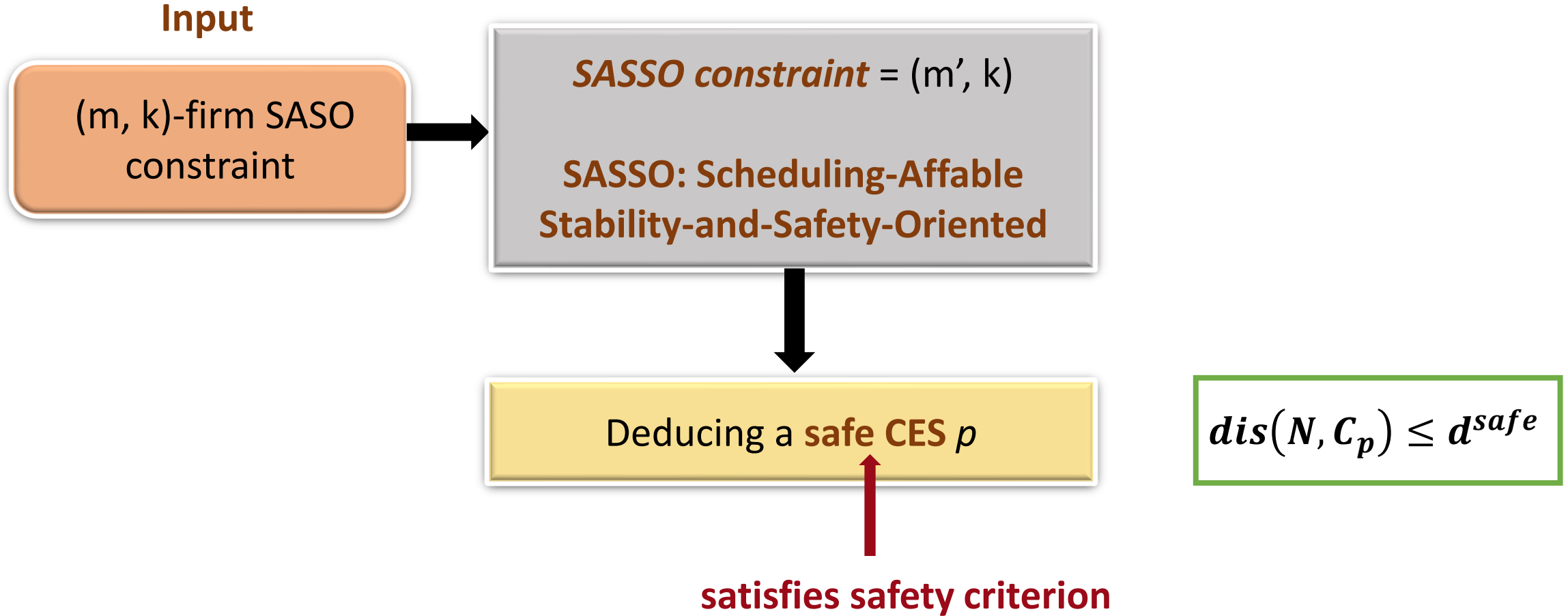
(m, k)-firm SASO
constraint



SASSO constraint = (m', k)

**SASSO: Scheduling-Affable
Stability-and-Safety-Oriented**

Ensuring Safety



Theoretical Results Established

1. A safe CES p corresponding to a SASSO constraint ensures control safety over an *unbounded time horizon*.

$$\text{dis}(N, C_P) \rightarrow 0 \text{ as } t \rightarrow \infty$$

Theoretical Results Established

1. A safe CES p corresponding to a SASSO constraint ensures control safety over an *unbounded time horizon*.
2. There exists an upper bound T_{ub} on the time horizon length for safety verification (i.e., checking $dis(N, C_p) \leq d^{safe}$).

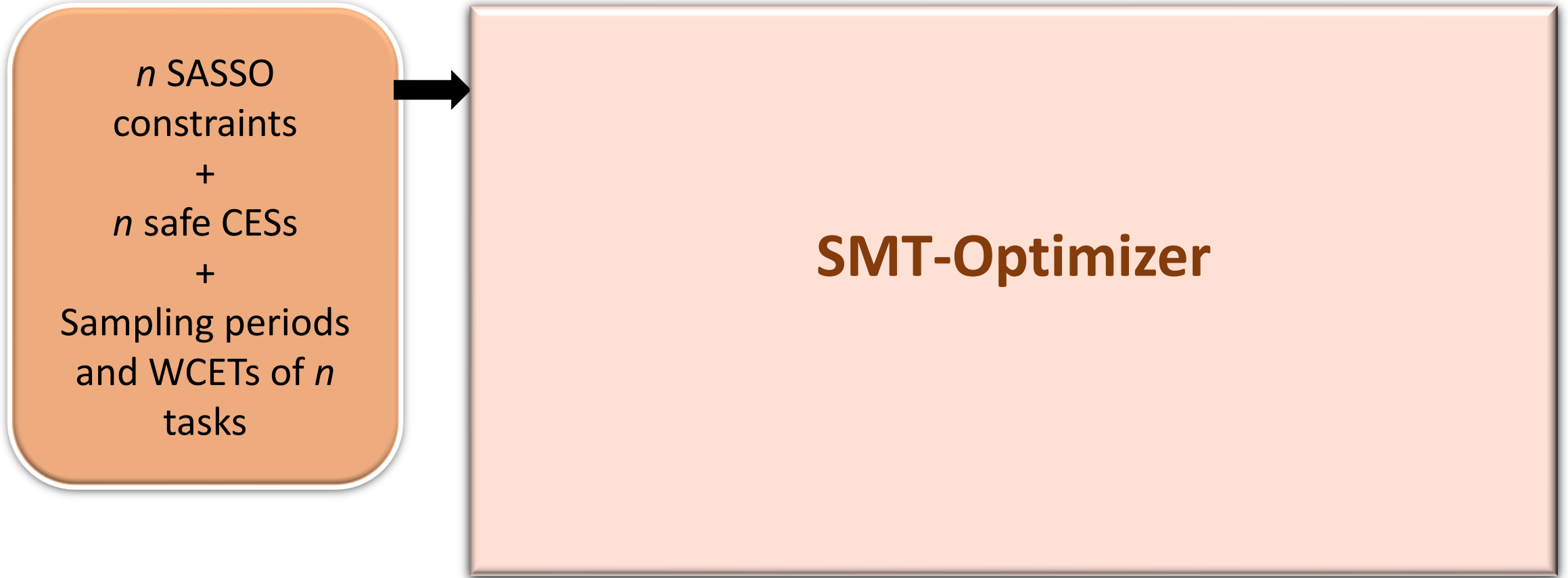
$$T_{ub} = \left\lceil \frac{\ln (2 ||\mathcal{A}_{ns}||^{l-1} ||x^n[0]||) - \ln (d^{safe})}{|\ln \epsilon|} \right\rceil \times l.$$

Theoretical Results Established

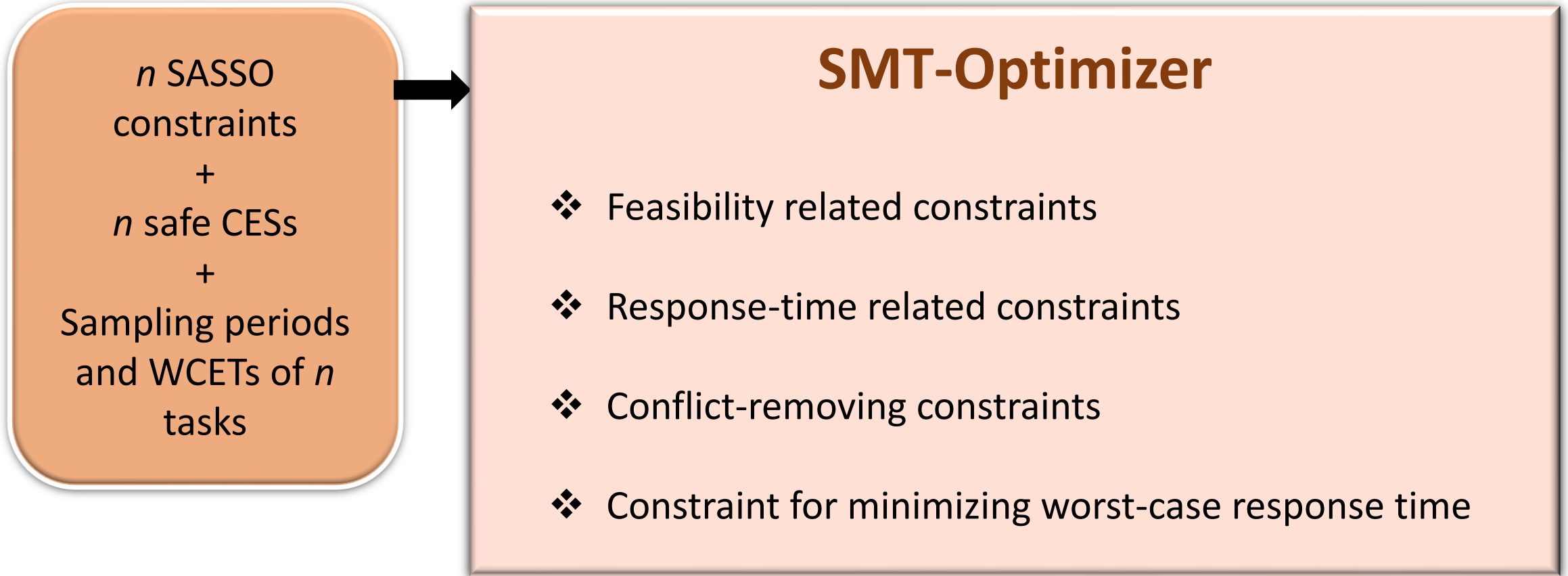
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2. There exists an upper bound T_{ub} on the time horizon length for safety verification (i.e., checking $dis(N, C_p) \leq d^{safe}$).
3. Exact time horizon length for safety verification is the *settling time*, from the time of application of an external disturbance.

Step 3: Schedulability Test and Synthesizing Schedule

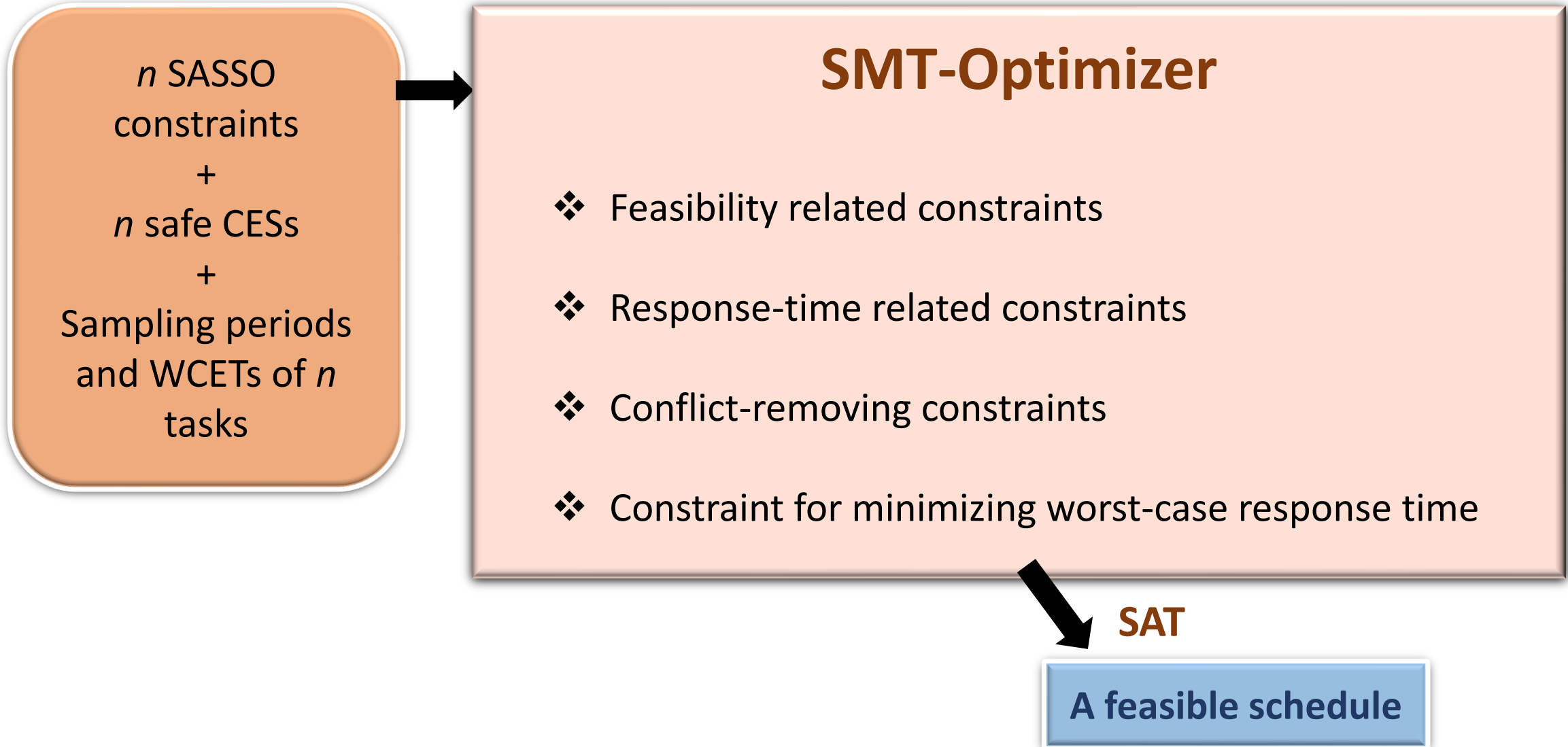
Schedulability Test and Synthesizing Schedule



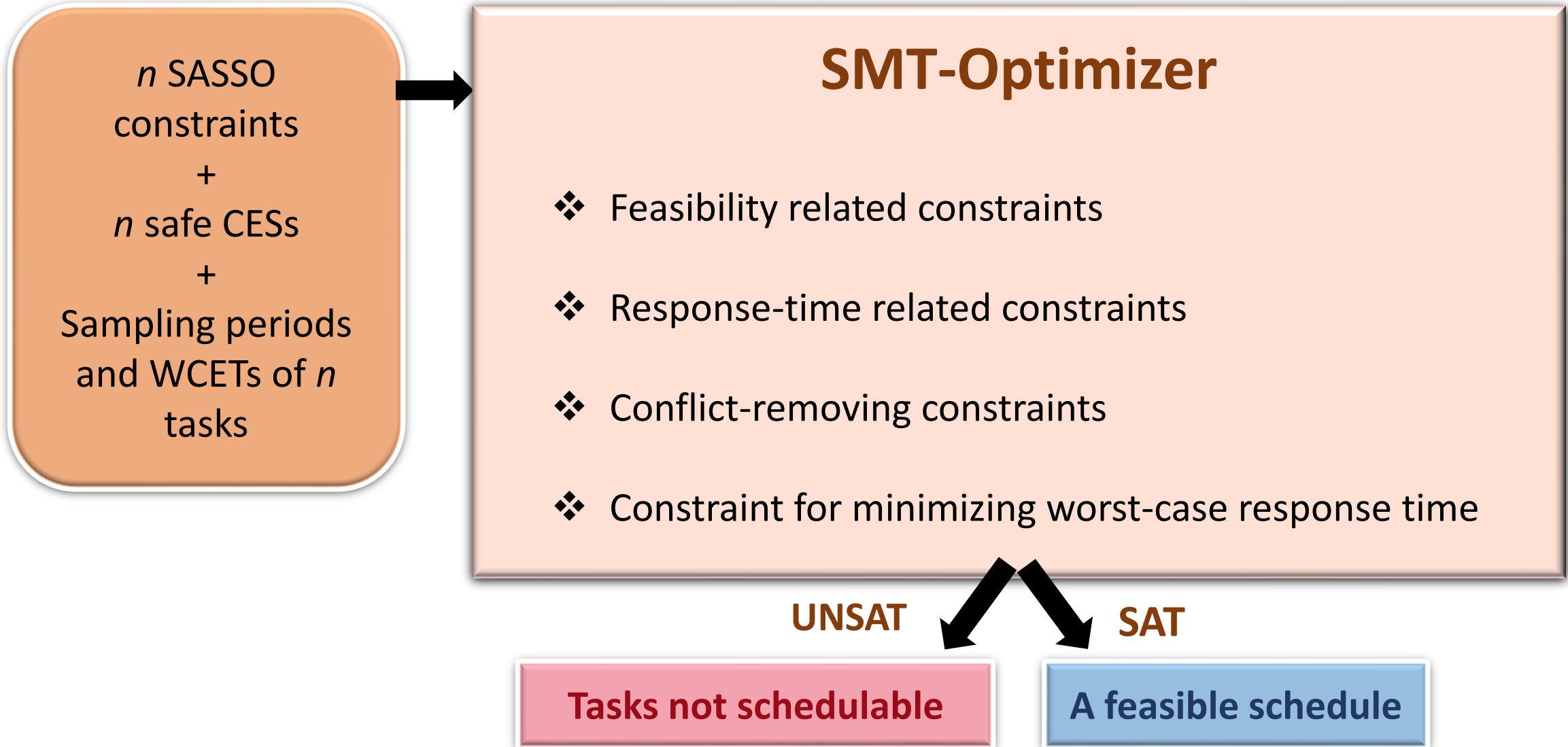
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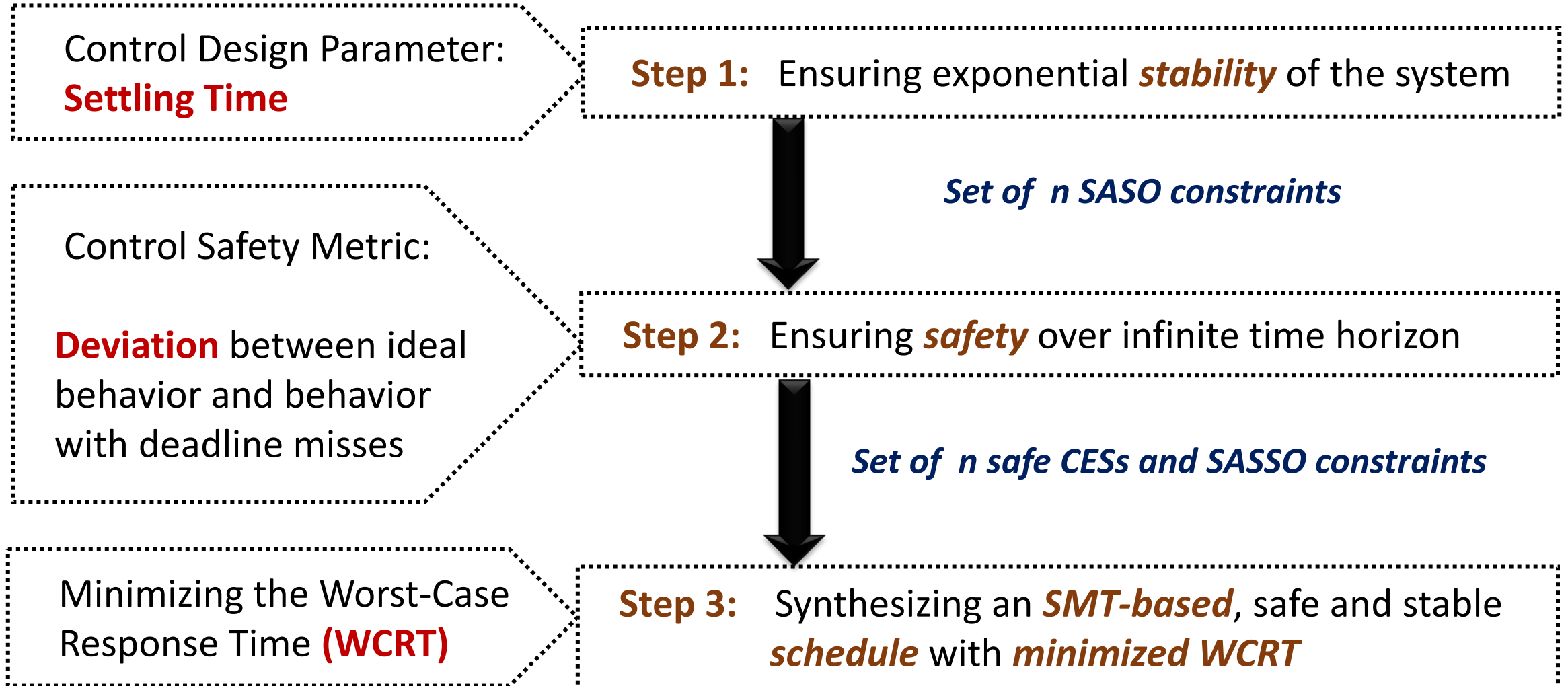
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Schedulability Test and Synthesizing Schedule



Summary of the Proposed Method

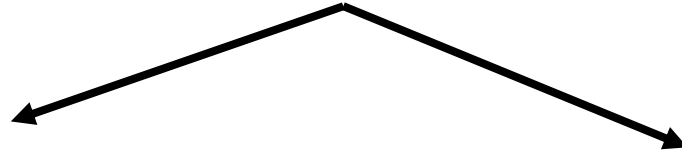


Evaluation

Sl. No.	Benchmark control systems considered from the automotive domain	Order of the system
1	Resistor-capacitor network (RC)	2
2	DC-motor speed control (DC)	2
3	Vehicle dynamic control (VDC)	2
4	Lane following Controller of an F1 tenth model car (F1)	2
5	Trajectory tracking control (TTC)	2
6	DC-servo control (DCS)	2
7	Cruise control (CC)	3
8	Adaptive cruise control (ACC)	3
9	Suspension control (SC)	4
10	Lane keeping system (LK)	4
11	Vision-based lateral control (LC)	5

Evaluation

2 parameters

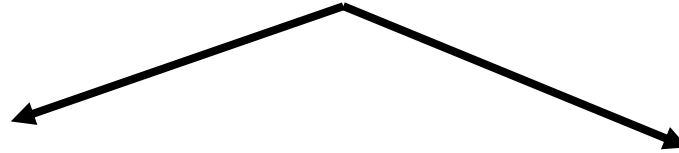


Number of plant-control systems
and corresponding control tasks

Processor utilization

Evaluation

2 parameters



Number of plant-control systems
and corresponding control tasks

Processor utilization

Tasks $\{T_i\}_{i=1 \text{ to } n} : T_i = \{(m'_i, k_i), p_i, c_i, h_i\}$

(m'_i, k_i) : SASSO constraint

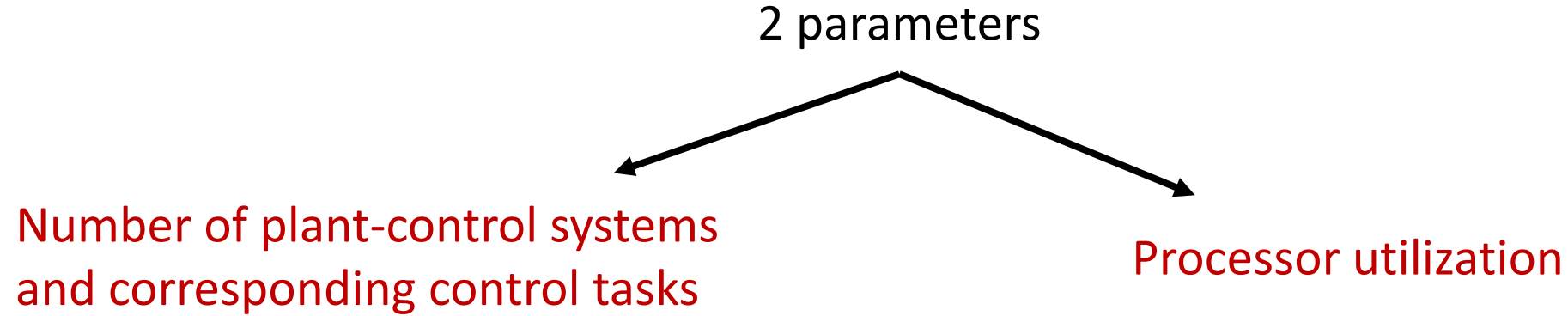
p_i : Safe CES

c_i : WCET of the control task

h_i : Sampling period of the controller

$$\text{Util.} = \frac{m'_i \times c_i}{k_i \times h_i}$$

Scalability Analysis



- ❖ *Increase* the number of **tasks** and **job instances**.
- ❖ *Increase* the **processor utilization** by *increasing WCET*.
- ❖ **Aim:** *Report a feasible schedule* with
 1. A *fairly reasonable runtime overhead*.
 2. *Improved* scope of *schedulability*.

Scalability Analysis

No of control tasks	No of jobs scheduled	Range of processor utilization for which a schedule is obtained	Time range to synthesize a safe and stable schedule
5	300	0.7 – 0.82	0.08 – 0.126 s (Less than 2s)
7	630	0.7 – 1.0	0.55 – 6.058 s (Less than 7s)
9	14,490	0.7 – 0.97	0.07 – 3.556 s (Less than 4s)
11	18,270	0.7 – 0.98	1.547 – 161.836 s (Less than 3min)
13	20,790	0.7 – 0.97	16.167 – 400.053 s (Less than 7min)
15	46,620	0.7 – 0.92	485.305 – 8517.031 s (Less than 2.5hr)

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- ❑ For 15 tasks, schedules are synthesized within 7 min-38 min up to Util. < 0.85.
- ❑ For Util. > 0.85, we obtain schedules but with considering a time-out of 3 hrs.

Comparison with the State-of-the-Art

State-of-the-art methods:

1) PGS (Pattern Guided Stable schedule)

- Sumana Ghosh, Souradeep Dutta, Soumyajit Dey and Pallab Dasgupta. 2017. *A Structured Methodology for Pattern Based Adaptive Scheduling in Embedded Control*. ACM Transactions on Embedded Computing Systems (TECS) 16, 5s, 189:1–189:22.

2) DSHT (Deterministic verification of schedule constructed with Statistical Hypothesis Testing)

- Shengjie Xu, Bineet Ghosh, Clara Hobbs, Enrico Fraccaroli, Parasara Sridhar Duggirala and Samarjit Chakraborty. 2023. *Statistical approach to efficient and deterministic schedule synthesis for cyber-physical systems*. In Proc. International Symposium on Automated Technology for Verification and Analysis (ATVA). Springer, 312–333.

3) SCS (Safe Constraint Synthesis to generate safe schedule)

- Shengjie Xu, Bineet Ghosh, Clara Hobbs, P. S. Thiagarajan and Samarjit Chakraborty. 2023. *Safety-aware flexible schedule synthesis for cyber-physical systems using weakly-hard constraints*. In Proc. Asia and South Pacific Design Automation Conference (ASP-DAC). 46–51.

Comparison with the State-of-the-Art

Our method referred to as **FMSS: Formal Methods for synthesizing a Safe and Stable schedule**

1. PGS (constructs a stable schedule)

Comparison with w.r.t. metrics:

- Safety
- Scope of schedulability
- Runtime Overhead

2. DSHT, SCS (construct a safe schedule)

Comparison with w.r.t. metrics:

- Stability
- Runtime Overhead

Comparison with PGS

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5
No of control tasks	Util.	Time taken by FMSS	Time taken by PGS stable constraints	Time taken by PGS with SASO constraints
5	0.76	0.100 s	80.625 s	0.456 s
	0.82	0.070 s	207.547 s	1.344 s
7	0.78	0.421 s	10.234 s	0.100 s
	0.82 – 1.0	✓	> 1 hr (timed out)	X
9	0.70	0.030 s	26.219 s	0.077 s
	0.72 – 0.97	✓	> 1 hr (timed out)	X
11	0.70 – 0.98	✓	> 1 hr (timed out)	X
13	0.70 – 0.92	✓	> 1 hr (timed out)	X
15	0.70 – 0.85	✓	> 1 hr (timed out)	X

1. Comparing runtime overhead

2. Comparing scope of schedulability

SASO constraints:
part of FMSS

Considered here for fair comparison

Comparison with PGS

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Improved runtime overhead in FMSS

Feasible schedule obtained within reasonable time

Suffers from time-out issues

Comparison with PGS

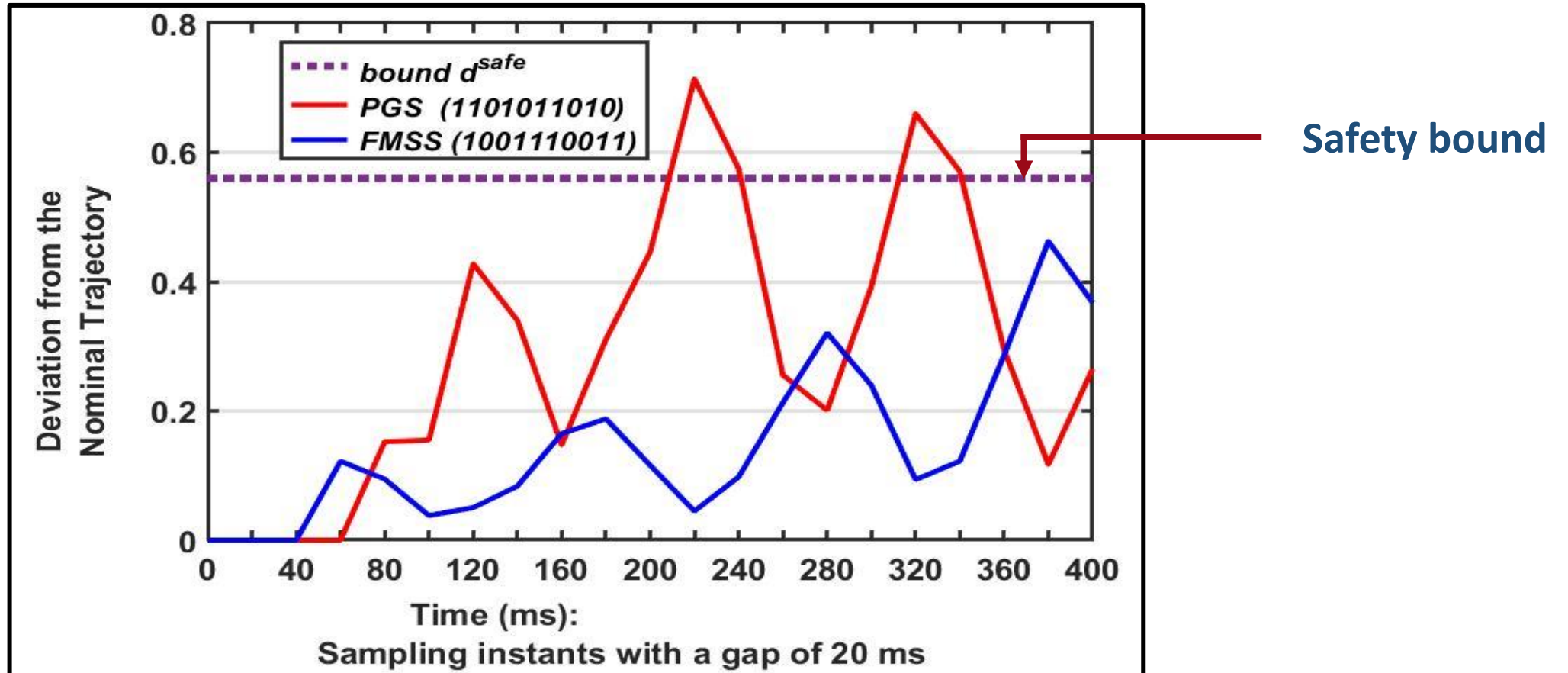
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Improved scope of schedulability in FMSS

Feasible schedule obtained

Tasks not schedulable

Comparison with PGS



Violation of safety bound in PGS in F1-tenth model car system

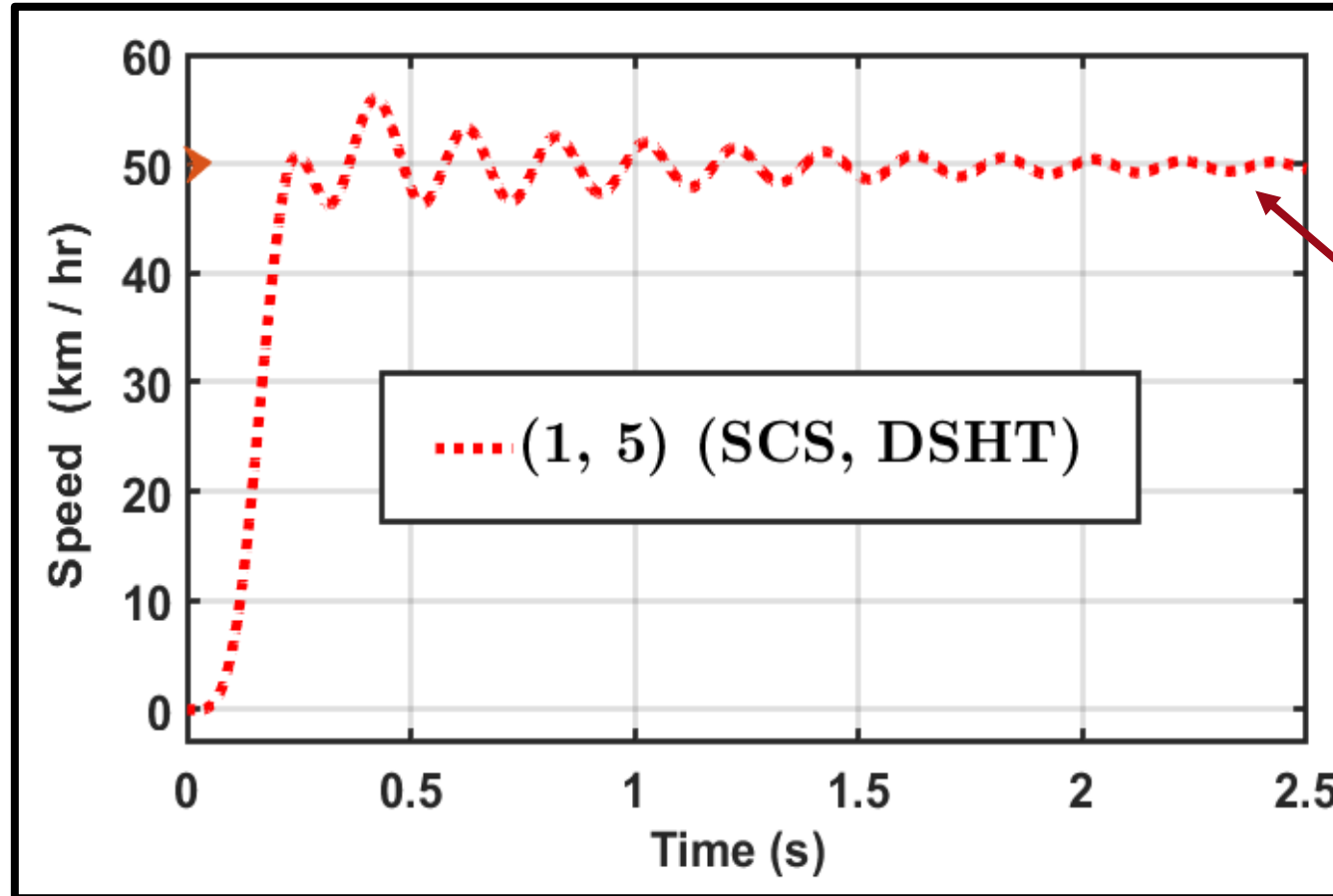
Comparison with DSHT and SCS

- ❖ We correlate the (l, ϵ) -exponential stability criterion with the *settling time*.

Settling time $\rightarrow$ (l, ϵ) -exponential stability criterion $\rightarrow$
stable constraint $\rightarrow$ SASSO constraint $\rightarrow$ *safe CES*.

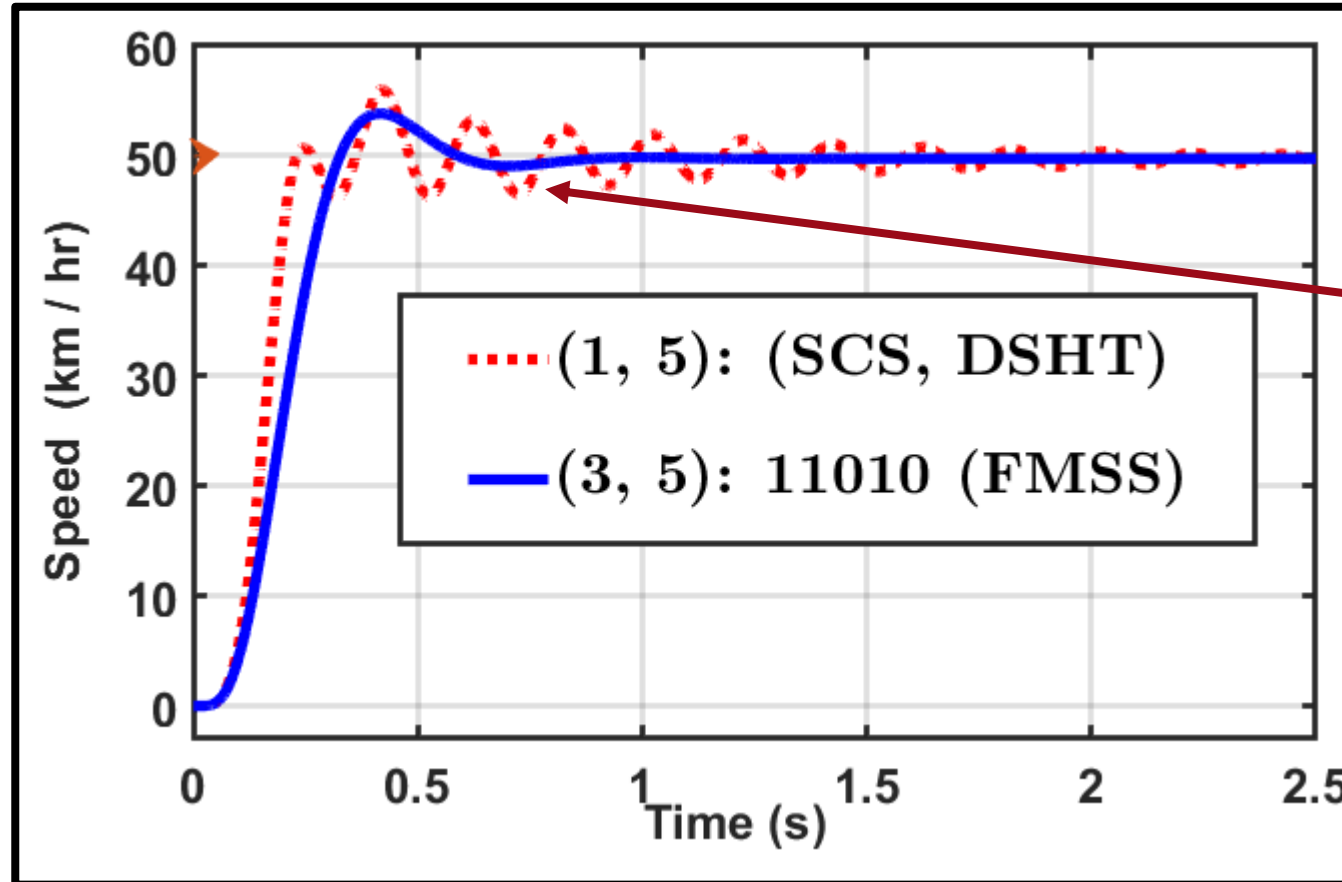
- ❖ A *shorter settling time* signifies a *stable* and a *more responsive* system.

Comparison with DSHT and SCS



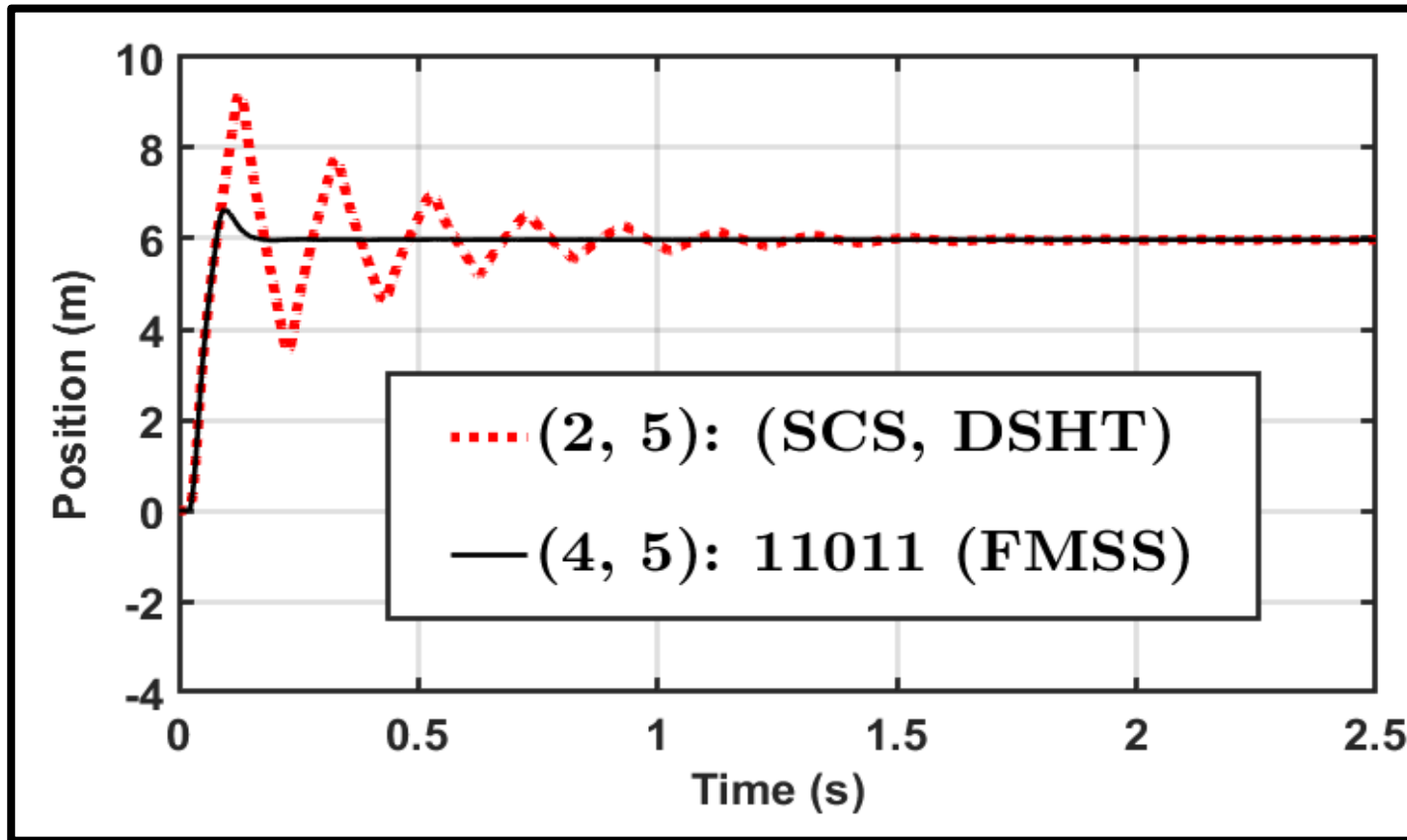
Cruise control system: (1, 5) is a safe constraint reported by SCS and DSHT

Comparison with DSHT and SCS



Cruise control system: (3, 5) is a SASSO constraint, 11010 is a safe CES, reported by FMSS

Comparison with DSHT and SCS



Settling time:
2.5s (DSHT, SCS)

Settling time:
0.2s (FMSS)

92% improvement
in FMSS

Suspension control system: (2, 5) is safe constraint reported by DSHT and SCS

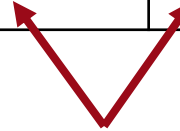
(4, 5) is a SASSO constraint, 11011 is a safe CES, reported by FMSS

Comparison with DSHT and SCS

Comparing the runtime overhead

No. of control tasks	Time taken (FMSS)	Time taken (DSHT)	Time taken (SCS)
5	0.08 s	13.50 s	748.37 s (12.47 min)

Runtime is quite high for a
small task setup



Comparison with DSHT and SCS

Comparing the runtime overhead

No. of control tasks	Time taken (FMSS)	Time taken (DSHT)	Time taken (SCS)
5	0.08 s	13.50 s	748.37 s

DSHT and SCS: employ **time-consuming** iterative approaches, probabilistic methods, reachability analysis

An exponential time rise possible for a higher number of tasks in DSHT and SCS !!

Comparative Comments on Other Works

Method in [1]:

- Constructs an SMT-based schedule.
- Ensures control safety in a weakly hard scenario.
- But the method is limited to medium-sized systems.

[1]: Anand Yeolekar, Ravindra Metta, and Samarjit Chakraborty. 2024. ***SMT-based Control Safety Property Checking in Cyber-Physical Systems under Timing Uncertainties***. In Proc. on VLSI Design and Embedded Systems (VLSID). 276–280.

Comparative Comments on Other Works

Comparing the runtime overhead

No. of control tasks	Time taken (FMSS)	Time taken [1]	Memory Consumed (FMSS)	Memory Consumed [1]
4	0.04 s	40 s	20 MB	48 MB

SMT-based scheduling in FMSS is **time-** and **compute-efficient**: pruned search space of the SMT-solver with SASO and SASSO constraints

Runtime and memory consumption is quite high for a small task setup

[1]: Anand Yeolekar, Ravindra Metta, and Samarjit Chakraborty. 2024. *SMT-based Control Safety Property Checking in Cyber-Physical Systems under Timing Uncertainties*. In Proc. on VLSI Design and Embedded Systems (VLSID). 276–280.

Conclusion

Contributions of this paper:

- ✓ ***Triplet (stability, safety, schedulability):*** Explored for the *first* time in the literature.
- ✓ ***Stability and safety:*** Ensured over an *infinite* horizon.
- ✓ ***Proposed scheduling approach:*** Minimized WCRT offers *improved scope of schedulability*.
- ✓ ***Proposed SMT-based approach:*** Time-efficient, hence *increases scalability*.
- ✓ ***Experimental comparison:*** Proposed method *outperforms the state-of-the-art* methods.

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Future works:

- Stability and safety for scheduling in *non-linear control systems*.
- *Dependencies in the task model* while designing a safe and stable schedule.

THANK YOU!